The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS & MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2018-2019

MECHANICS OF SOLIDS

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL questions in Section A and THREE questions in Section B

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

In this examination candidates are required to answer ALL questions in Section A and THREE out of FIVE questions in section B. If a candidate answers more than the required number of questions, all questions will be marked and the highest marks will be used in the final examination mark.

ADDITIONAL MATERIAL: Multiple choice answer sheet
Formula sheet
Answer booklet

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

SECTION A
Answer ALL questions in this section

1. What is the value of the maximum in-plane principal stress for the 2D plane-stress element shown in Figure Q1?

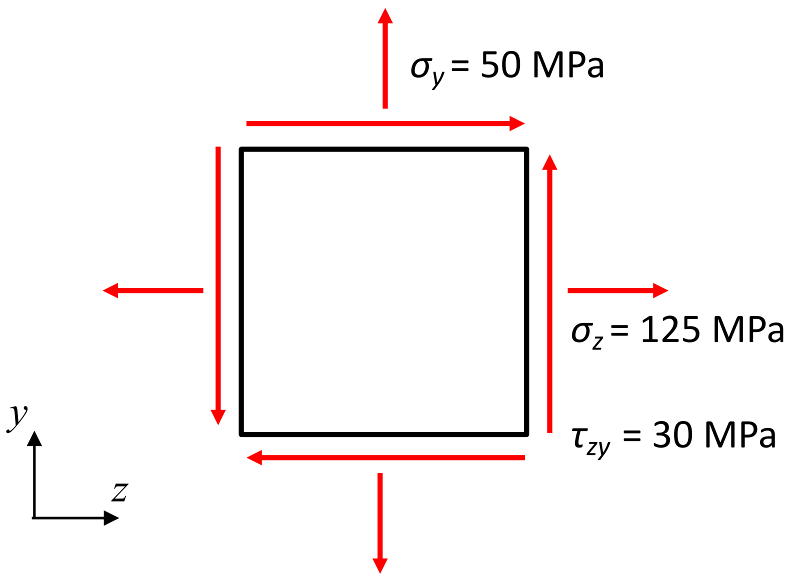

Figure Q1

A. 92.5MPa
B. 87.5 MPa
C. 48 MPa
D. 135.5 MPa
E. 85.5 MPa

[2]

2. In Figure Q2, regions A and B represent regions of:

A. Yielding and buckling, respectively
B. Buckling and fatigue, respectively
C. Cracking and buckling, respectively
D. Buckling and yielding, respectively
E. Fatigue and buckling, respectively

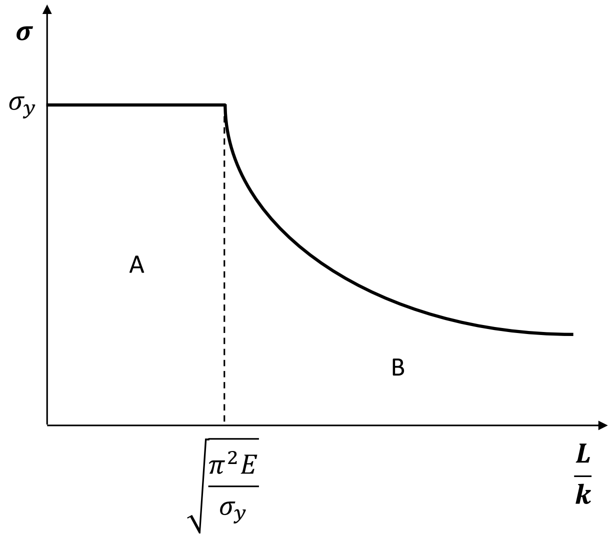

Figure Q2

where E and σ_y are the Young's modulus and yield stress of the material, $\frac{L}{k}$ is the slenderness ratio of the beam. [2]

3. Figure Q3 illustrates which type of material behaviour?

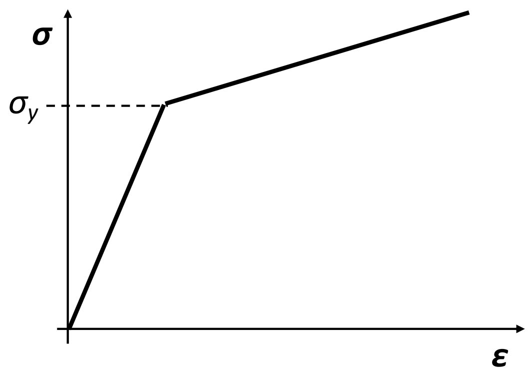

Figure Q3

A. elastic-perfectly-plastic
B. linear softening
C. non-linear elasticity
D. non-linear hardening
E. linear hardening

[2]

4. A bar with a rectangular cross-section 20 mm wide and 40 mm deep is subjected to a vertical shear force of 20 kN. What is the value of maximum shear stress in the bar?

A. 25 MPa
B. 37.5 MPa
C. 75 MPa
D. 18.75 MPa
E. 56.25 MPa

[2]

5. A thick-walled cylinder with a 25 mm bore and a 50 mm OD is internally pressurised at 250 bar. What are the values of Lamé's constants for this problem?

A. $A = -\frac{625}{75}$, $B = -\frac{62500}{3}$
B. $A = \frac{625}{75}$, $B = \frac{62500}{3}$
C. $A = -\frac{15625}{1875}$, $B = -\frac{15625}{3}$
D. $A = \frac{625}{75}$, $B = -\frac{15625}{3}$
E. $A = \frac{15625}{1875}$, $B = \frac{15625}{3}$

[2]

6. The following expression describes the strain energy in a beam:

$$U = \frac{P^2}{EI}\left(\frac{L^3}{6} + \frac{3\pi L^2 R}{2} + \frac{4\pi R^3}{5} + \pi R^2\right)$$

What is the deflection at the position of and in the direction of load, P?

A. $u = \frac{P^3}{3EI}\left(\frac{L^3}{6} + \frac{3\pi L^2 R}{2} + \frac{4\pi R^3}{5} + \pi R^2\right)$

B. $u = \frac{P^2}{EI}\left(\frac{L^4}{24} + \frac{3\pi L^3 R}{6} + \left(\frac{4\pi R^3}{5} + \pi R^2\right) L\right)$

C. $u = \frac{2P}{EI}\left(\frac{L^3}{6} + \frac{3\pi L^2 R}{2} + \frac{4\pi R^3}{5} + \pi R^2\right)$

D. $U = \frac{P^2}{EI}\left(\frac{L^3}{6} + \frac{3\pi L^2 R}{2} + \frac{4\pi R^3}{5} + \pi R^2\right)$

E. $U = \frac{P^4}{12EI}\left(\frac{L^3}{6} + \frac{3\pi L^2 R}{2} + \frac{4\pi R^3}{5} + \pi R^2\right)$

[2]

7. The hydrostatic stress component in Figure Q7 is indicated by:

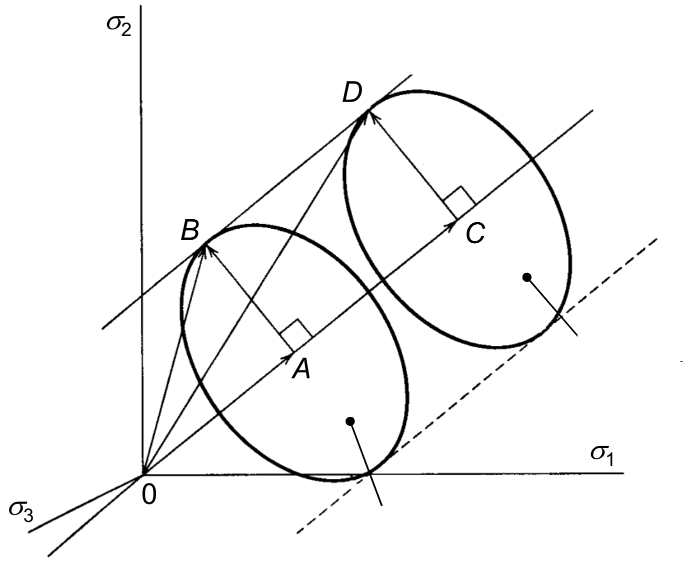

Figure Q7

- **A.** OA
- **B.** OB
- **C.** AB
- **D.** OD
- **E.** CD

where σ_1, σ_2 and σ_3 are the three principal stresses. [2]

8. If a beam has a rectangular cross-section, as shown in Figure Q8, has a yield stress, σ_y, of 215 MPa, and is subjected to a pure bending moment of 200 kNm, does yielding occur?

Assume elastic-perfectly-plastic material behaviour.

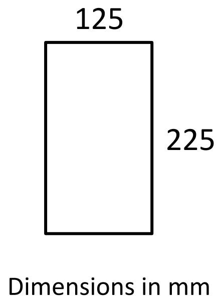

Figure Q8

A. Yes
B. -
C. -
D. -
E. No

[2]

9. The stiffness matrix for a one-dimensional spring element is given by:

A. $\begin{bmatrix} -k & k \\ k & -k \end{bmatrix}$

B. $\begin{bmatrix} -k & -k \\ -k & k \end{bmatrix}$

C. $\begin{bmatrix} -k & -k \\ -k & -k \end{bmatrix}$

D. $\begin{bmatrix} k & k \\ k & k \end{bmatrix}$

E. $\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$

[2]

10. Calculate the position of the centroid, C, for the beam cross-section shown in Figure Q10.

A. $\bar{x} = 20\ mm$, $\bar{y} = 30\ mm$
B. $\bar{x} = 0\ mm$, $\bar{y} = 0\ mm$
C. $\bar{x} = 9\ mm$, $\bar{y} = 22.67\ mm$
D. $\bar{x} = 10\ mm$, $\bar{y} = 30\ mm$
E. $\bar{x} = 9\ mm$, $\bar{y} = 24.5\ mm$

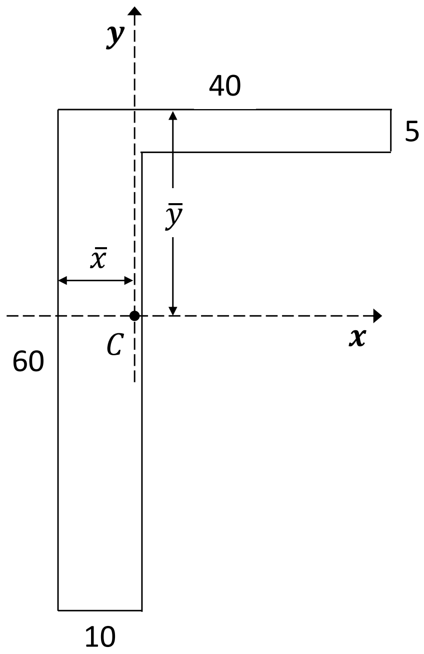

Figure Q10

[2]

11. In the prediction of fatigue life, the Gerber curve is:

A. More conservative than the Goodman line
B. Also known as the Goodman line
C. Less conservative than the Goodman line
D. The same as the Goodman line in terms of conservatism
E. Not comparable

[2]

12. For the beam cross-section shown in Figure Q12, the shear centre will be located closest to which point?

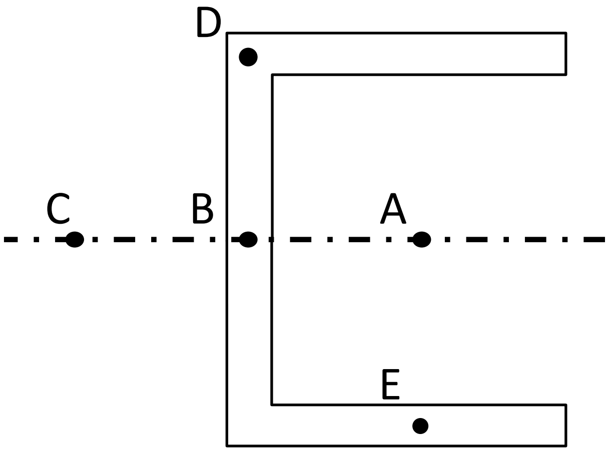

Figure Q12

A. A
B. B
C. C
D. D
E. E

[2]

13. A 40 mm diameter solid bar will carry an axial load of 40 kN and a torque of 500 Nm, what is the maximum in-plane shear stress on a 2D plane stress element on the surface of the bar?

A. 17 MPa
B. 40 MPa
C. 43 MPa
D. 51 MPa
E. 80 MPa

[2]

14. A rotor disc with an angular velocity of 2000 rpm has an external diameter of 0.8 m and has a 0.1 m diameter hole bored along its axis. What is the value of hoop stress at the bore? (ρ = 7900 kgm^{-3}, v = 0.3).

A. 46 MPa
B. 63 MPa
C. -63 MPa
D. 72 MPa
E. -46 MPa

[2]

15. What is the limiting pressure to avoid yielding for a steel pressure vessel 2 m in diameter and 5 mm thick according to the von Mises yield criterion?
(σ_y = 250 MPa).

A. 0.95 MPa
B. 1.44 MPa
C. 1.37 MPa
D. 1.90 MPa
E. 2.88 MPa

[2]

16. The translation of the yield surface shown in Figure Q16 represents

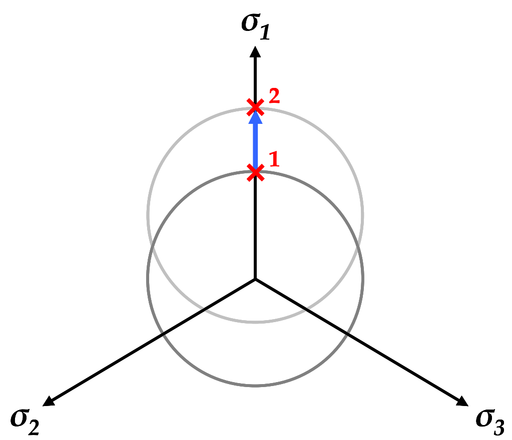

Figure Q16

A. elastic-perfectly-plastic material behaviour
B. isotropic hardening
C. linear softening
D. kinematic hardening
E. non-linear softening

[2]

17. If a beam under bending has the following 2nd order differential equation:

$$EI\frac{d^2y}{dx^2} = R_A x + M_O\langle x-3\rangle^0 - w\frac{\langle x-6\rangle^2}{2}$$

What is the corresponding expression for slope of the beam?

A. $EI\frac{dy}{dx} = R_A\frac{x^2}{2} + M_O\langle x-3\rangle - w\frac{\langle x-6\rangle^3}{6}$

B. $y = \frac{1}{EI}\left(R_A\frac{x^3}{6} + M_O\frac{\langle x-3\rangle^2}{2} - w\frac{\langle x-6\rangle^4}{24} + Ax + B\right)$

C. $\frac{dy}{dx} = \frac{1}{EI}\left(R_A\frac{x^2}{2} + M_O\langle x-3\rangle - w\frac{\langle x-6\rangle^3}{6} + A\right)$

D. $EIy = R_A\frac{x^3}{6} + M_O\frac{\langle x-3\rangle^2}{2} - w\frac{\langle x-6\rangle^4}{24}$

E. $\frac{dy}{dx} = \frac{1}{EI}\left(R_A\frac{x^3}{6} + M_O\frac{\langle x-3\rangle^2}{2} - w\frac{\langle x-6\rangle^4}{24} + A\right)$

[2]

18. A square bar (a = 22 x 10^{-6}, E = 70 GPa) with cross-section 20 mm x 20 mm is fixed rigidly between two walls at 70 °C. When the temperature is reduced to 20 °C, what is the resultant stress in the bar?

A. 72 MPa
B. -77 MPa
C. -72 MPa
D. 77 MPa
E. 108 MPa

[2]

19. The expression for critical buckling load of a fixed-fixed beam is:

A. $P_c = \frac{\pi^2 EI}{4L^2}$

B. $P_c = \frac{\pi^2 EI}{L^2}$

C. $P_c = \frac{2.045\pi^2 EI}{L^2}$

D. $P_c = \frac{\pi^2 EI}{2.045L^2}$

E. $P_c = \frac{4\pi^2 EI}{L^2}$

where EI is the flexural rigidity and L is the length of the beam. [2]

20. The expression for strain energy in a beam under bending is:

A. $U = \int_0^L \frac{P^2}{2AE} ds$

B. $U = \int_0^L \frac{M^2}{2EI} ds$

C. $u = \frac{\partial U}{\partial P}$

D. $U = \int_0^L \frac{T^2}{2JG} ds$

E. $u = \int_0^L \frac{T^2}{2EI} ds$

where P is load, M is bending moment and T is torque. [2]

SECTION B

Answer THREE questions in this section

21. Two cylinders of equal length have the following dimensions:

    i. 10 mm bore and 20.02 mm outside diameter
    (steel: E = 210 GPa, ν = 0.3, α = 12 × 10^{-6})

    ii. 20 mm bore and 40 mm outside diameter
    (bronze: E = 100 GPa, ν = 0.3, α = 17 × 10^{-6})

    The larger cylinder is heated, placed around and allowed to shrink onto the smaller cylinder.

    (a) Determine the minimum temperature increase required to allow assembly. [5]

    (b) Determine the interface pressure after assembly. [15]

22. (a) Describe with the aid of diagrams the relationship between crack growth rate and stress intensity factor. [4]

    (b) Define by means of an equation the stress intensity factor and give examples of the effects of finite boundaries on stress intensity factors. [4]

    (c) Show with the aid of diagrams the effects of mean stress on fatigue crack growth. [4]

    (d) A large steel plate with an edge crack, for which $K_I = 1.12\sigma\sqrt{\pi a}$, has a fracture toughness, $K_{I_{cr}}$, of $175 MPa\sqrt{m}$ and a yield stress, σ_y, of $210 MPa$. If the applied stress, σ, is $\frac{3}{5}\sigma_y$, determine the critical crack size assuming linear elastic material. [8]

23. A beam with the cross-section shown in Figure Q23 for which the direction of the principal axes and the magnitudes of the principal 2nd moments of area are as given below, is subjected to a bending moment, $M = 2000Nm$ along the x-axis.

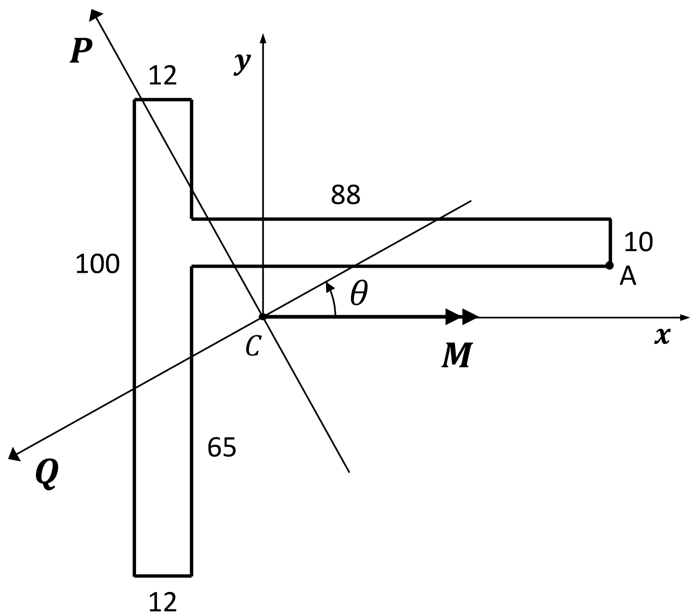

Figure Q23

$$\theta = 28.87°$$
$$I_P = 2.131 \times 10^6\ mm^4$$
$$I_Q = 9.305 \times 10^5\ mm^4$$

Determine:

(a) The position/orientation of the neutral axis. [7]

(b) The bending stress at position A. [7]

(c) Location and value of the maximum compressive bending stress. [6]

24. Figure Q24a shows a right-angle beam, built into the floor at one end and subjected to a point load, P, at the other, free end. The beam is made from steel bar, circular in cross section, with a diameter, ϕ, of 50 mm, a Young's Modulus, E, of 209 GPa and a Shear Modulus, G, of 72 GPa.
The beam initially stands on the $x-y$ plane at a distance, a, of 75 mm, from a nearby parallel wall, as shown in Figure Q24a. P is applied in the z-direction and has a magnitude of 2.5 kN. Both straight regions of the beam are 1 m in length. Figure Q24b shows the beam and wall viewed on the $x-z$ plane.

Does the free tip of the bracket make contact with the wall? Show your working, using Strain Energy, to determine the answer.

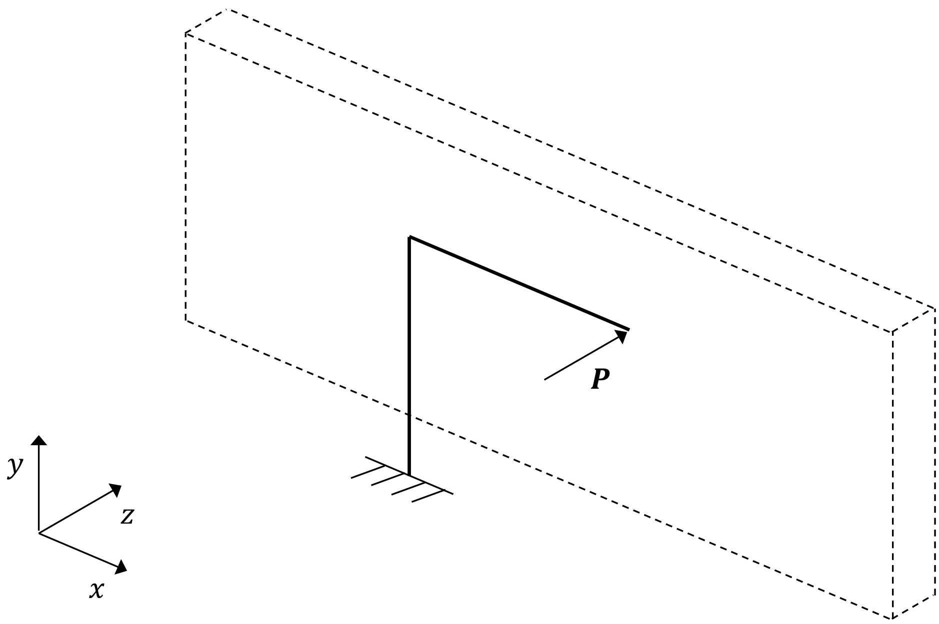

Figure Q24a

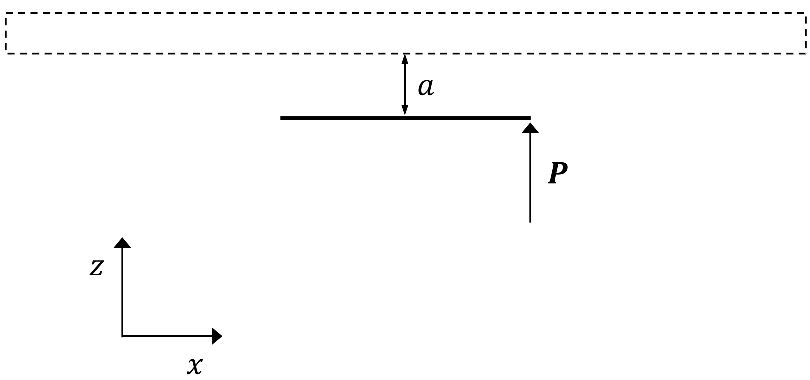

Figure Q24b

[20]

25. The pin-jointed framework ABC is subjected to an external load, $F = 20kN$, as shown in Figure Q25. If each member has a length of 600 mm, a cross-section of 20 x 30 mm^2 and is made of aluminium with a value of Young's modulus of 70 GPa:

(a) Construct the stiffness matrix of the structure. [12]

(b) Determine the horizontal and vertical components of displacement at point B. [8]

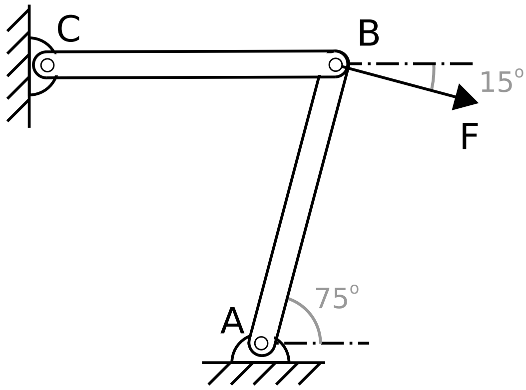

Figure Q25

END